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NUCLEAR STRUCTURE PROBLEMS IN DOUBLE BETA DECAY

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ABSTRACT

We summarize some of the nuclear structure issues in theoretical treatments of $\beta\beta$ decay, emphasizing several of the less well-tested approximations commonly made.

INTRODUCTION

The recent attempts to extract quantitative limits on lepton number violation from double beta decay experiments have underscored the need for reliable treatments of the nuclear structure of this process. In this talk we discuss our present understanding of the nuclear physics, the approximations that are commonly made by structure theorists, and the prospects for improving existing calculations. Because of the excellent talk by Professor Kotani, we will not discuss the particle physics aspects of this process nor its importance as a constraint on gauge theories. We will also not present numerical limits on lepton number violation: these results are contained in recent reviews.[1,2,3]

DECAY RATES

In deriving the decay rate for the 2ν $\beta\beta$ decay process shown in Fig. 1a, two approximations are commonly made:

i) Each nuclear β decay is evaluated in the allowed approximation where only the Fermi and Gamow-Teller operators ($\tau_+(i)$ and $\vec{\sigma}(i)\tau_+(i)$) are retained;

ii) The sum over virtual intermediate nuclear states is performed by closure after replacing the nuclear excitation energy appearing in the energy denominator by an average value.

The first approximation restricts the states populated in the daughter nucleus by the decay of a 0^+ parent to those with $J^\pi=0^+,1^+$, and 2^+. In fact, decays to 1^+ and 2^+ states are strongly suppressed. Decays between 0^+ states are mediated by two matrix elements

$$M_{GT} = \langle 0_f^+ | \tfrac{1}{2} \sum_{ij} \vec{\sigma}(i)\cdot\vec{\sigma}(j)\tau_+(i)\tau_+(j) | 0_i^+ \rangle \qquad (1a)$$

$$M_F = \langle 0_f^+ | \tfrac{1}{2} \sum_{ij} \tau_+(i)\tau_+(j) | 0_i^+ \rangle \qquad (1b)$$

The double Fermi matrix element vanishes in the limit of good isospin and quite generally is small. Thus the decay rate can be written approximately as

$$\omega_{2\nu} \sim f_{GT} |M_{GT}|^2 \qquad (2)$$

where the phase space factor f_{GT} is

$$f_{GT} \sim \frac{2m_e^{11}(G \cos\theta_c)^4}{\pi^7 7!} \; \frac{F^2(z)}{\langle E\rangle^2} \; [\tilde{T}_o^7 + \ldots + \frac{\tilde{T}_o^{11}}{1980}]$$

with θ_c the Cabibbo angle, F(Z) a correction for distortions of the electron plane waves in the Coulomb field of the nucleus, <E> the average intermediate state excitation energy, and $\tilde{T}_o$ the total kinetic energy carried off by the leptons in units of $m_e c^2$.

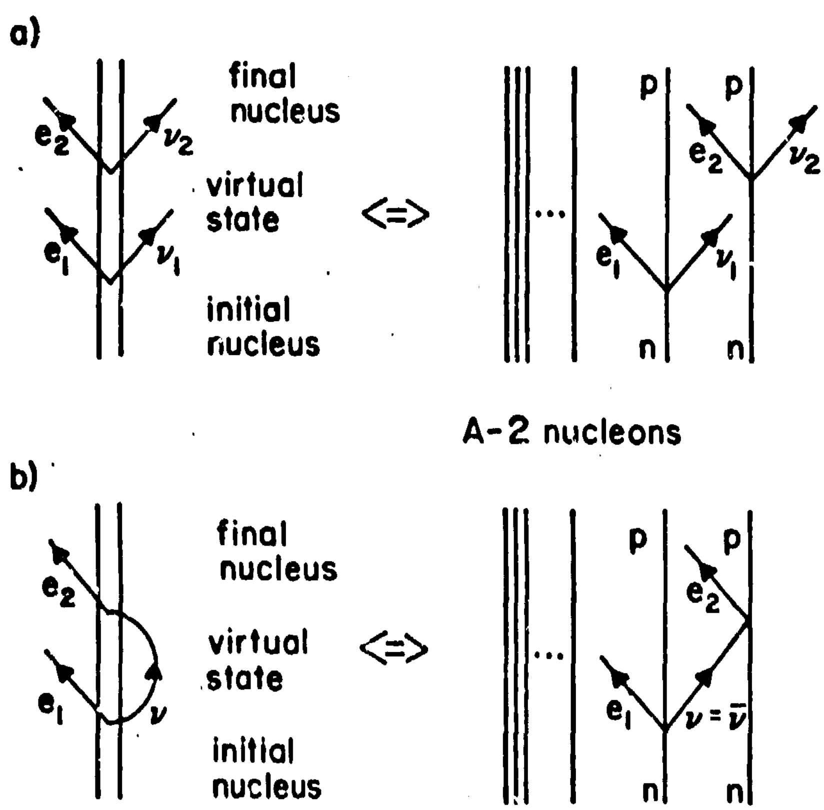

Fig. 1. Two-nucleon mechanisms for (a) two-neutrino and (b) no-neutrino $\beta\beta$ decay.

The analogous calculation of 0ν $\beta\beta$ decay is more complicated. First, as this process does not occur in the standard model, we must introduce a more general β decay Lagrangian. One form commonly used allows left- and right-handed leptonic and hadronic currents.[1,2] The left- and right-handed neutrino fields are massive. More correctly, they can be expressed in terms of the 2n Majorana mass eigenstate fields, where n is the number of generations $(e,\mu,\tau,\ldots)$ in some underlying theory. If the Lagrangian is CP-invariant and all neutrino mass eigenstates are light, the 0ν $\beta\beta$ decay rate depends quadratically on lepton-number-violating masses and couplings such as

$$\langle m_\nu \rangle_{LL} = \sum_{i=1}^{2n} \lambda_i^{CP} |U_{ei}^L|^2 \, m_i$$

and

$$\eta_{RL} \langle 1 \rangle_{LR} \text{ with } \langle 1 \rangle_{LR} = \sum_{i=1}^{2n} \lambda_i^{CP} \, U_{ei}^L U_{ei}^R \tag{3}$$

Here U_{ei}^L and U_{ei}^R are the coefficients describing the left- and right-handed electron neutrinos in terms of the mass eigenstates, and m_i and λ_i^{CP} are the mass and CP eigenvalue of the ith mass eigenstate. The coefficient η_{RL} is the strength of the coupling of the right-handed leptonic current to the left-handed hadronic current in units of the weak coupling G_F. Both η_{RL} and m_i destroy the γ_5-invariance (i.e., maximal parity violation) of the standard model. In addition to nonzero values for η_{RL} and m_i, the sums appearing in eqs. (3) must be nonvanishing if 0ν $\beta\beta$ decay is to occur. The presence of Majorana terms in the neutrino mass matrix will produce nonvanishing sums. More details on the particle physics can be found elsewhere.[1,2,3]

We retain only the Gamow-Teller and Fermi matrix elements and electron s- and p-waves. The closure approximation is invoked to complete the sum over virtual intermediate states. (This approximation is better justified than in 2ν decay because of the presence of an energetic virtual neutrino in the intermediate state.) One finds that the 0ν decay rate for $0^+ \to 0^+$ transitions depends on a variety of nuclear matrix elements.

$$\langle 0_f^+ | \tfrac{1}{2} \sum_{ij} \tau_+(i)\tau_+(j) \, \vec{\sigma}(i)\cdot\vec{\sigma}(j) \, \frac{g(r_{ij})}{r_{ij}} \, | 0_i^+ \rangle \tag{4a}$$

$$\langle 0_f^+ | \tfrac{1}{2} \sum_{ij} \tau_+(i)\tau_+(j) \, \frac{g(r_{ij})}{r_{ij}} \, | 0_i^+ \rangle \tag{4b}$$

$$\langle 0_f^+ | \tfrac{1}{2} \sum_{ij} \tau_+(i)\tau_+(j) \, \hat{r}_{ij}\cdot\vec{\sigma}(i) \, \hat{r}_{ij}\cdot\vec{\sigma}(j) \, \frac{g(r_{ij})}{r_{ij}} \, | 0_i^+ \rangle \tag{4c}$$

$$\langle 0_f^+ | \tfrac{1}{2} \sum_{ij} \tau_+(i)\tau_+(j)\hat{R}_{ij} \cdot (\hat{r}_{ij} \times (\vec{\sigma}(i)-\vec{\sigma}(j))) \frac{g(r_{ij})R_{ij}}{r_{ij}^2} \; 0_i^+ \rangle \qquad (4d)$$

where $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ and $\vec{R}_{ij} = \vec{r}_i + \vec{r}_j$. One can take $g(r_{ij}) \sim 1$ for light neutrinos ($m_\nu \ll 1/R_0$, with R_0 the nuclear radius). As only the first two matrix elements contribute to terms in the decay rate proportional to $\langle m_\nu \rangle_{LL}^2$, in this limit there is a close analogy between the matrix elements governing 0ν and 2ν decay.

The quality of the constraints we can place on the parameters governing lepton number violation depends on our ability to calculate these 0ν matrix elements. The similarity between the 0ν matrix elements and those mediating 2ν decay suggests a natural check on the reliability of such calculations: do the nuclear wave functions reproduce known 2ν decay rates? A comparison of shell model predictions of Haxton, Stephenson, and Strottman[4] with experiment is made in Table 1. Limits on $|M_{GT}|$ from laboratory experiments with ^{48}Ca and ^{82}Se are somewhat smaller than the calculated values. More serious disagreements exist between theory and geochemical half life determinations for ^{82}Se, ^{128}Te, and ^{130}Te.

Table 1. Calculated and experimental double Gamow-Teller matrix elements M_{GT}.

Nucleus	$\lvert M_{GT}\rvert$ theory[4]	$\lvert M_{GT}\rvert$ exp*
^{48}Ca	0.22	<0.20[5]
^{76}Ge	1.28	
^{82}Se	0.94	<0.75[6]
		0.40[7]†
^{128}Te	1.47	0.21-0.25[8]†
		<0.19[7]†
^{130}Te	1.48	0.11-0.14[7,9]†
		0.19[8]

*Calculated using $F_A = 1.25$, $\cos_c = 0.9737$, and $M_F = 0$.

†Maximum values determined from total geochemical rates.

NUCLEAR STRUCTURE ISSUES

The limits on lepton number violation that can be extracted from bounds on 0ν $\beta\beta$ decay rates are given in Ref. 1. We will not repeat those discussions here, but rather concentrate on the general nuclear structure issues. Clearly the credibility of any limit on lepton number violation depends on our confidence in matrix element calculations, and that confidence would be enhanced if we understood the origin of the discrepancies between theory and the geochemical 2ν $\beta\beta$ decay results. Thus we review several of the less well-tested approximations that are commonly made in nuclear structure studies of $\beta\beta$ decay.

1. Completeness of the model space: The double Gamow-Teller operator is simple in that $\vec{\sigma}(i)\tau_+(i)$ does not change the nodal or orbital quantum numbers of a nucleon. In principle, this allows one to respect exactly the underlying "sum rule" governing this operator: bases can be defined such that the Gamow-Teller operator cannot cause transitions to configurations outside the model space.

In some early calculations this sum-rule constraint was rather badly violated: basis truncations were so severe that virtually none of the configurations produced when $\vec{\sigma}(i)\cdot\vec{\sigma}(j)\tau_+(i)\tau_+(j)$ acts on the initial state were included in the description of the final state (and conversely). This, and the use of nonrelativistic electron wave functions, led to long half lives, in agreement with the geochemical results.

In the shell model calculations that produced the results in Table 1, great effort was expended to satisfy the sum rule constraint. The tendency of minor components of the wave functions to contribute constructively to M_{GT} then leads to "large" matrix elements in all cases except ^{48}Ca. We believe we understand this behavior: Zamick and Auerbach showed that this constructive addition of strengths of different orbitals is exact in the asymptotic limit of the Nilsson model.[10] The size of $|M_{GT}|$ is governed largely by the pairing force: a stronger pairing force permits greater overlaps between neutron and proton configurations.

However, some compromises are necessary even in large-basis shell model calculations. In our Te calculations we employed the canonical shell model space spanning the magic numbers 50 and 82, $3s_{1/2}$-$2d_{3/2}$-$2d_{5/2}$-$1g_{7/2}$-$1h_{11/2}$. This space omits the spin partners $1g_{9/2}$ and $1h_{9/2}$ and is thus incomplete in the sense of the sum rule. Inclusion of the missing spin partners makes the shell model diagonalization much more difficult and also produces a spurious (center-of-mass) model space. This spuriosity is a serious difficulty in calculations where one uses realistic g-matrix interactions that are not translationally invariant. Despite the shell model prejudice that the missing orbitals are not as important as those we have included, it would clearly be better to include them. A perturbative estimate of the contributions of the missing spin partners could be made without running into difficulties with spurious center-of-mass effects.

2. Completion of the sum over intermediate nuclear states by closure: The matrix element M_{GT} is derived by replacing the 1/E-weighted sum over virtual nuclear states by a non-weighted sum so that closure can be used

$$\frac{1}{2}\sum_n \frac{\langle f|\sum_i \vec{\sigma}(i)\tau_+(i)|n\rangle\langle n|\sum_j \vec{\sigma}(j)\tau_+(j)|i\rangle}{E_i - \varepsilon - \nu - E_n} = \frac{M_{GT}}{\langle E_i - E_n\rangle - \varepsilon - \nu} \qquad (5)$$

Although this can be regarded as a definition of the average excitation energy, in practice $\langle E_i - E_n\rangle$ is taken from (p,n) mappings of the single Gamow-Teller strength distributions in the intermediate nu leus. However, if the signs of the terms in Eq. (5) have predominantly one value for small E_n and the opposite value for large E_n, this procedure could give a result that differs significantly from the 1/E-weighted sum.

A number of tests of the closure approximation have been made. In many cases these tests were done by severely restricting the model space, so that explicit summations over the intermediate states in Eq. (5) could be performed. We believe such tests are inadequate because of the need for realistic model spaces discussed previously. An RPA summation[11] over intermediate states in ^{130}I and a partial summation over low-lying states in ^{82}Br are described in Ref. 1; both tend to support the closure approximation. More recently Tsuboi, Muto, and Horie[12] performed a shell model summation over intermediate states excited in the $\beta\beta$ decay of ^{48}Ca, the simplest of the isotopes of Table 1. The energy-weighted sum gave a decay rate substantially larger than the closure result and larger than the experimental limit established by the Columbia group.[5] However the ^{48}Ca $\beta\beta$ decay matrix element M_{GT} is unusual, strongly suppressed by cancellations occurring in the $f_{7/2}$ shell because of the K selection rule discussed by Lawson[13] and by Zamick and Auerbach.[13] In the presence of such cancellations it is not surprising that the 1/E-weighted and nonweighted sums produce different results. It is unclear whether any conclusions can be drawn about the other $\beta\beta$ decay nuclei of Table 1, where no strong cancellations occur in the shell model estimates of M_{GT}.

Explicit summations over intermediate states can be extremely difficult numerically. A more tractable alternative may be to construct

$$\sum_n \frac{|n\rangle\langle n|\sum_j \vec{\sigma}(j)\tau_+(j)|i\rangle}{E_i - \varepsilon - \nu - E_n} \qquad (6)$$

from the moments H^m of the distribution $\Sigma\vec{\sigma}(j)\tau_+(j)|i>$, where H is the nuclear Hamiltonian. A large number of moments can be calculated by techniques used routinely in shell model codes employing the Lanczos algorithm.[14] Alternatively, the first moment of H can be expressed as a double commutator and could be evaluated in most model calculations.

3. The allowed approximation: Our expression for the 2ν $\beta\beta$ decay rate (Eq. (2)) was derived in the long-wavelength approximation while retaining only the leading operators in (v/c) of the nucleon. The long-wavelength approximation is well justified because the momentum transfer to the nucleus is restricted by the lepton kinematics to $<<1/R_o$, where R_o is the nuclear radius.

The neglect of $\vec{p}/M$ terms in the weak hadronic current is less well justified. The most important of these operators may be the axial charge, $\vec{\sigma}(i)\cdot\vec{p}(i)/M$, which carries odd parity. As a pseudoscalar, it will not couple single-particle states in the Te model space described previously. (There is no j=11/2 even-parity partner of the $h_{11/2}$ orbital.) However, principal components of the Te ground state will connect to minor components of the Xe ground state outside this model space, and conversely. The axial charge operator could prove more important for heavier nuclei, where valence protons and neutrons predominantly occupy orbitals of opposite parity.

4. Delta-hole components in nuclear wave functions: As the delta can be produced when the Gamow-Teller operator acts on a nucleon, our sum-rule argument also suggests one should consider Δ-h components in the nuclear wave functions. Despite the large energy required to produce this resonance, Δ-h excitations can be important because there is no Pauli blocking. The role of Δ-h excitations in single β decay is still controversial because nuclear correlations may account for much of the observed Gamow-Teller quenching.[15] Grotz and Klapdor[16] found a 25-30% reduction in $\beta\beta$ decay rates due to Δ-h components.

5. Quality of the nuclear structure calculations: Finally, apart from all of the approximations described above, there remains the question of how well we can execute nuclear structure calculations. The results reported in Table 1 are taken from "state-of-the-art" shell model calculations involving very large bases. However, some compromises for the sake of numerical simplicity had to be made, the principal one being a weak coupling approximation (see Ref. 1).

One should bear in mind, in comparing different calculations, that one can reproduce observed 2ν $\beta\beta$ decay rates by adjusting the effective interaction: turning off the pairing interaction greatly reduces $|M_{GT}|$. In the shell model calculations of Ref. 4, no such ad hoc adjustments were made: g-matrices generated from realistic

N-N interactions were used. The goal of future nuclear structure studies of 2ν $\beta\beta$ decay should be to reproduce observed rates while using models and effective interactions consistent with our understanding of nuclear structure.

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